

# The Exceptional Lie symmetry groups hierarchy and the expected number of Higgs bosons

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## Abstract

New insights into the structure of various exceptional Lie symmetry groups hierarchies are utilized to shed light on various problems pertinent to the standard model of high energy physics and the Higgs.

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## 1. Introduction

From a historical viewpoint, it could be said that modern quantum mechanics was invented three times using three different formalisms [1–3]. The first is matrix theory which was largely due to Heisenberg [2]. The second, namely wave mechanics is due to Schrödinger [1]. Lastly the third is quantum algebra which was introduced by Dirac [3]. From this perspective it may seem that a fourth formulation using geometry and number theory was overdue.

In retrospect,  $E$ -Infinity theory [4–11] is what comes closest to such formulation utilizing the geometry of numbers and the numeric of geometry to give a more comprehensive insight into the structure and topology of the quantum world [4–11].

In the spirit of the above, the present paper looks closely at abstract symmetry groups and in particular the exceptional  $E_i$  Lie groups hierarchy to shed new light on the  $E$ -Infinity approach to particle physics [7,8]. Proceeding this way, we will discuss the particle content of a modestly extended standard model and give a rational explanation to the expected number of eight Higgs bosons [8].

## 2. The Anti de Sitter space

The Maldacena conjecture suggests that string theory on  $AdS_5$  is equivalent to a super-symmetric Yang Mills theory on the boundary manifold. Since  $AdS_5$  is an unwrapped  $1 + 4$  dimensional anti de Sitter space and  $S^5$  is a space-like 5 sphere then the total number of dimensions may be regarded as  $9 + 1$  rather than simply 10 dimensions [5,9]. Next we look at the dimension of the corresponding Riemann curvature for  $n = 9$

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$$R^{(9)} = (n^2)(n^2 - 1)/12 = (9)^2(9^2 - 1)/12 = 540$$

Now let us introduce the usual E-Infinity three-step symmetry breaking and find

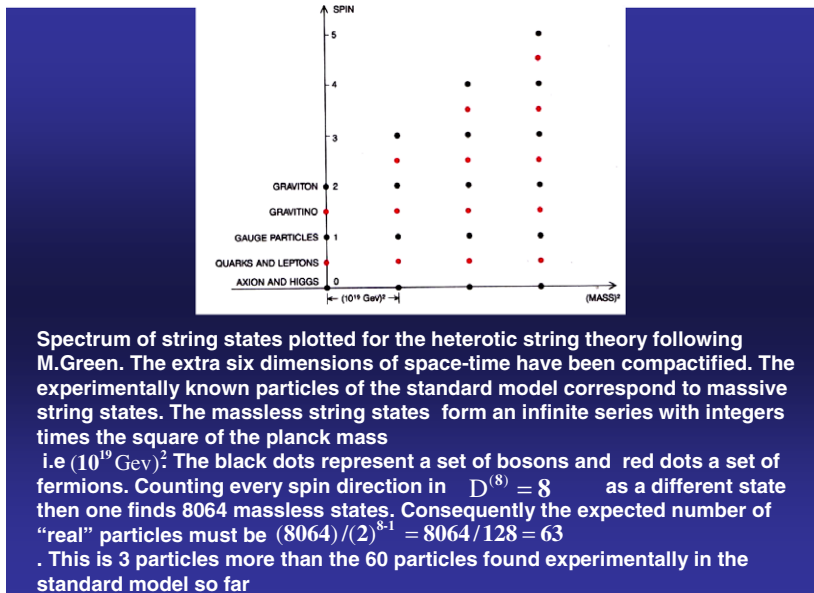
$$N = R^{(9)}/8 = 67.5$$

However in proceeding this way, we must take the additional one dimension representing originally the time. To take it into account would naively lead to  $N = 68.5$  which would correspond to the total number of isometrics equal to  $540 + 8 = 548$  and consequently to

$$N = 548/8 = 68.5$$

This result is very close indeed to the exact expectation value of the number of elementary particles namely [9]

$$N(\text{SM}) = (26 + k)(336 + 16k)/128 + 8k = \bar{\alpha}_0/2 = \mathbf{68.5}$$



Note further that using the instanton number of K3 namely  $n=24$  and the holographic boundary isometries of Klein modular curve i.e  $|SL(2,7)| = 336$  one finds the same result  $(24)(336) = 8064$ . The exact result was found by El naschie to be  $(26+K)(336+16K) = 8872.135957$  leading to  $8872.135957 / (128+8K) = 68.54101967$  which is the number predicted by E-infinity theory

Fig. 1.

where  $26 + k$  is the number of instantons of a fuzzy K3 Kahler manifold,  $336 + 16$  is the number of isometrics on the holographic boundary and  $128 + 8k$  is the transfinite value of the number of spin directions (see Fig. 1). In other words, the exact number of isometrics in a transfinite  $AdS_5 \otimes S^5$  is not 540 or 548 but  $548 + 4k_0$  which means  $(4)(\bar{\alpha}_0)$  where  $\bar{\alpha}_0 = 137 + k_0$  is the theoretical fine structure constant of electro-magnetism [5,9].

### 3. The exceptional Lie group hierarchy

As we reasoned in earlier publications, the hierarchy of space time is reflected into a deep symmetry group hierarchy connected to the  $E_i$  line of the exceptional Lie groups. In particular, after identifying  $SO(10)$  with  $E_5$  and  $SU(5)$  with  $E_4$  one finds that [8]

$$\sum_{i=4}^{i=8} |E_i| = \mathbf{24} + \mathbf{45} + \mathbf{78} + \mathbf{133} + \mathbf{248} = 528$$

Surprisingly, this is exactly equal to the number of brane state for  $D = 11$  and  $P = 5$  namely [9]

$$N^{(11)}(\text{Brane}) = \mathbf{11} + \mathbf{55} + \mathbf{462} = 528$$

Or equally, the number of killing vector fields for  $D = (4)(8) = 32$

$$N_{(k)}^{(32)} = (n)(n+1)/2 = (32)(33)/2 = 528$$

However we could extend the argument by including  $|SU(3)SU(2)U(1)| = \mathbf{12}$  of the standard model and write that [8,9]

$$\sum_{i=4}^{i=8} |E_i| + |SU(3)SU(2)U(1)| = \mathbf{528} + \mathbf{12} = 540$$

which is exactly equal to  $R^{(9)}$ .

Now we could argue based on the above that the additional 8 needed for 540 to be 548 is nothing but an additional color group  $|SU(3)| = \mathbf{8}$  or more convincingly an additional eight degrees of freedom connected to the illusive Higgs field. It is remarkable that purely abstract symmetry group structure could correspond not only to an alleged space time structure but to a fundamental physical concept such as the Higgs [11].

### 4. Relation between exceptional Lie groups and the 69 standard model particles prediction of E-Infinity

It is well known that E-Infinity theory predicted a maximum number of  $\bar{\alpha}_0/2 \simeq \mathbf{69}$  elementary particles in a consistently extended standard model. In fact we have derived recently the following relationship [7–9]

$$\sum_{i=4}^{i=8} |E_i| / (D^{(8)}) = \mathbf{528}/\mathbf{8} = |F_4| + |G_2| = 52 + 14 = 66$$

which links all the standard exceptional Lie groups  $E_8, E_7, E_6, F_4$  and  $G_2$  together.

For the non-standard Lie group  $E_5$  and  $E_4$ , on the other hand one notices immediately that [6,9]

$$|E_5| + |E_4| = |SO(\mathbf{10})| + |SU(\mathbf{5})| = 45 + 24 = 69$$

Continuing this result transfinitely one finds that

$$45 \Rightarrow (2)(21 + k) = 2(26 + k - 45) = 42 + 2k$$

and

$$24 \Rightarrow (2)(13 + k/2) = 26 + k$$

In other words, we have

$$|E_5|_c + |E_4|_c \Rightarrow (\mathbf{42} + \mathbf{2k}) + (\mathbf{26} + k) = 68.54101967 = \bar{\alpha}_0/2 \cong \mathbf{69}$$

### 5. The fundamental equation of the holographic boundary

The fundamental equation relating the isometrics of the bulk to that of gravity, strong force, weak force and electromagnetism was found in earlier publications to be [8]

$$\mathbf{B} = \mathbf{H} + \mathbf{G} + \mathbf{E}$$

where  $\mathbf{B}$  is the bulk,  $\mathbf{H}$  is the total sum of isometrics representing the particle physics which is living on the holographic boundary,  $\mathbf{G}$  is Einstein’s space in four dimensions or Kaluza–Klein gravity in five dimensions and  $\mathbf{E}$  represents the isometrics of electromagnetism. The situation is graphically explained in Figs. 1–3. We see for instance that in case of superstrings theory we have

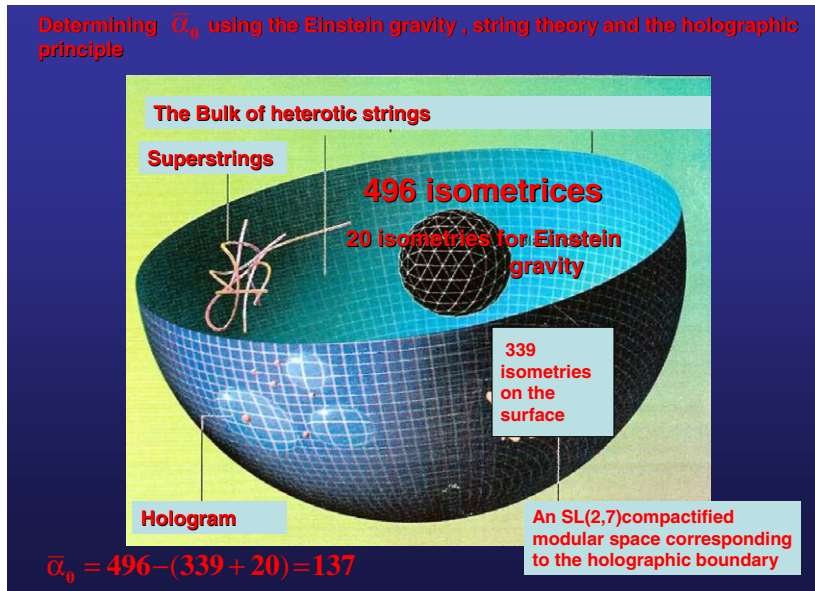


Fig. 2. Holographic principle applied to string theory.

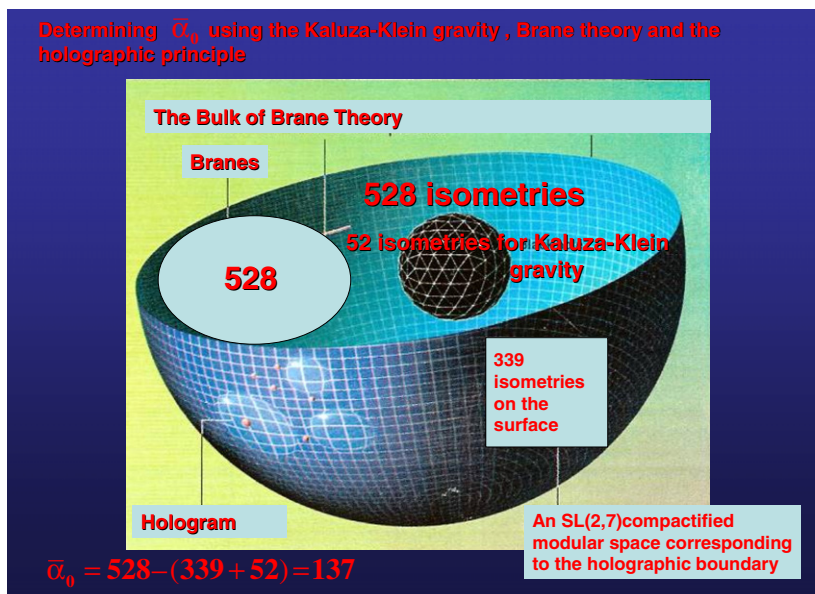


Fig. 3. Holographic principle applied to brane theory.

$$\begin{aligned}
 B &= |E_8 E_8| - k^2 \cong \mathbf{496} \\
 H &= |\mathrm{SL}(2, 7)| + \mathbf{16}k = \mathbf{339} \\
 G &= R^{(4)} = 20
 \end{aligned}$$

Consequently one finds that

$$496 = 339 + 20 + E$$

Thus

$$E = 496 - 339 - 20 = 137 = \bar{\alpha}_0$$

where  $k = \phi^3(1 - \phi^3)$  and  $\bar{\alpha}_0$  is the inverse electromagnetic fine structure constant.

On the other hand, when using the  $p = 5$  brane theory in the 11-dimensional  $M$  theory we have

$$\begin{aligned}
 B &= 528 \\
 H &\cong 339
 \end{aligned}$$

and

$$G \simeq 50 + 2 = 52$$

Consequently, one finds

$$E = 528 - 339 - 52 = 137 = \bar{\alpha}_0$$

In Figs. 2 and 3 we show some of the possible applications of our fundamental equation using different theories.

### 6. Conclusion

The exceptional Lie symmetry group hierarchy possesses miraculous properties which could be explained rationally as a consequence of the hierarchy of  $E$ -Infinity space time. To start with, the sum of the  $E_i$  dimensions is equal to the number of the killing vector fields in super space given by  $n = (4)(8) = 32$ . In other words, we have

$$\sum_{i=4}^{i=8} |E_i| = \mathbf{24} + \mathbf{45} + \mathbf{78} + \mathbf{133} + \mathbf{248} = 528 = (32)(32 + 1)/2 = K^{(32)}$$

Taking the standard model  $\mathrm{SU}(3)\mathrm{SU}(2)\mathrm{U}(1)$  as the next subgroup in the hierarchy, one finds

$$K^{(32)} + \mathrm{SU}(3)\mathrm{SU}(2)\mathrm{U}(1) = 528 + 12 = 540$$

Consequently, we may conclude that the 8 needed to reach 548 isometrics is the same 8 needed for  $R^{(9)}$  to come to the same value for the 9 + 1 dimensional anti de Sitter space of the Maldacena conjecture.

In addition, we have made it clear that the exceptional  $F_4$  as well as  $G_2$  maybe continued transfinitely to give

$$|F_4| = \mathbf{52} \Rightarrow |F_4|_c = \left(\frac{\bar{\alpha}_0}{\mathbf{10}}\right)(\mathbf{4}) = \mathbf{54.83281572}$$

and

$$|G_2| = \mathbf{14} \Rightarrow |G_2|_c = \bar{\alpha}_0/\mathbf{10} = \mathbf{13.70820393}$$

Therefore we have

$$|F_4|_c + |G_2|_c = \bar{\alpha}_0/2 \cong \mathbf{69}$$

Similarly, we may proceed using

$$E_5 = \mathrm{SO}(\mathbf{10}) \quad \text{and} \quad E_4 = \mathrm{SU}(\mathbf{5})$$

and find that

$$|E_5| = \mathbf{45} \Rightarrow |E_5|_c = \mathbf{42} + \mathbf{2}k$$

and

$$|E_4| = \mathbf{24} \Rightarrow |E_4|_c = \mathbf{26} + k$$

Therefore, we obtain

$$|E_5|_c + |E_4| = (42 + 2k) + (26 + k) = \bar{\alpha}_0/2 \simeq 69$$

In conclusion, we looked once more at the fundamental equation

$$\mathbf{B} = \mathbf{H} + \mathbf{G} + \mathbf{E}$$

Using different theories, we have demonstrated the power of the holographic principle in conjunction with  $E$ -Infinity theory to determine  $\bar{\alpha}_0 \simeq 137$  and give it a geometrical meaning.

## References

- [1] Jammer Max. The philosophy of quantum mechanics. New York: Wiley; 1974.
- [2] Heisenberg W. The physical principles of quantum theory. New York: Dover; 1930.
- [3] Dirac PAM. The principles of quantum mechanics. Oxford University Press; 1982.
- [4] El Naschie MS. Elementary prerequisites for  $E$ -Infinity. Chaos, Soliton & Fractals 2006;30(3).
- [5] El Naschie MS. On D Gross'criticism of S. Eddington and an exact calculation of  $\bar{\alpha}_0 \simeq 137$ . Chaos, Soliton & Fractals 2007;32:1245–9.
- [6] El Naschie MS. A review of  $E$ -Infinity theory and the mass spectrum of high energy particle physics. Chaos, Soliton & Fractals 2004;19:209–36.
- [7] El Naschie MS. Symmetry group prerequisites for  $E$ -Infinity in high energy physics. Chaos, Soliton & Fractals, in press. doi:10.1016/j.chaos.2007.05.006.
- [8] El Naschie ME. Noether's theorem, Exceptional Lie group hierarchy and determining  $1/\bar{\alpha}_0 = 137$  of electromagnetism. Chaos, Soliton & Fractals, in press. doi:10.1016/j.chaos.2007.05.005.
- [9] El Naschie MS. A review of applications and results of  $E$ -Infinity theory. Int J Nonlinear Sci Numer Simulat 2007;8(1):11–20.
- [10] Je-Huan He, Sigalotti LDG, Mejias A. Beyond the 2006 physics nobel prize for COB E. Shanghai, PR China. ISBN 988-97681-9-4/0.4.
- [11] Ji-Huan He, Transfinite physics. Shanghai, PR China. ISBN 988-98846-5-8, 2005.