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## Strange non-dissipative and non-chaotic attractors and Palmer's deterministic quantum mechanics

The present letter was prompted by some very recent realization of the deep meaning of relatively well established facts connected to the foundation of quantum mechanics and the mathematical physics of chaos and fractals.

For the sake of lucidity and to make our letter reasonably self contained despite space limitation, we have to regress in time and start around 1995 when an important book [1] edited by Nobel laureate Prigogine, Rössler and El Naschie appeared with what was at the time the unusual title 'Quantum Mechanics, Diffusion and Chaotic fractals'. With a great deal of justification, this book could be said to have marked a new area where the new scientific revolution of nonlinear dynamics, chaos and fractals was systematically integrated with general relativity and quantum mechanics [2]. The main aim of this effort was the development of a general theory of quantum gravity [3,4]. To achieve this aim it was clear that quantum mechanics must first be freed from many paradoxes and mysteries and what could be more mysterious and paradoxical than the outcome of the two-slit experiment with quantum particles? One of the earliest attempts to solve this problem using nonlinear dynamics and fractals is in two papers by El Naschie in the above mentioned book [1]. The papers were also published earlier on in Chaos, Solitons & Fractals (see Ref. [5] and the references therein). The main idea of El Naschie's work at that early stage was to establish a connection between the three and four points chaos game [5–7] and the two-slit experiment. In particular it was shown that the appearance of the Sierpinski triangle in the three points version of the chaos game is equivalent to the invariant limit set of the game [4,7]. In other words, the Sierpinski gasket is the strange attractor of this game. Since it is meaningless to talk about any kind of dissipation in this context, then this attractor is non-dissipative attractor although it has a fractal structure [5,7]. In other words we could speak somewhat loosely of a Hamiltonian strange attractor or a non-dissipative strange attractor. This initially seemingly paradoxical notion is not as outlandish as it may seem. In fact we will reveal in a moment that non-dissipative strange attractors occur generically and are called in topology, the VAK which is an acronym for the vague attractor of Kolmogorov. The VAK was conjectured to be the invariant limiting set of the stationary states of quantum mechanics by Thom [8] and was integrated into high energy physics for the first time by El Naschie [4,9].

Leaving this development for a moment, we would like to draw attention to an important paper by a strong Belgian group of nonlinear dynamics led by Nicolis and Nicolis [5]. In this paper a highly original connection was established between El Naschie's idea of explaining the two-slit experiment using fractals in the form of iterated fractal functions and the riddled fractal basin boundaries of an iterated dynamical system [5]. It seems that this three Nicolis paper inspired a distinguished British scientist, Palmer who like Nicolis works in a meteorological research institute but is seriously interested in the foundations of quantum mechanics [10]. There was a big problem however. To connect to quantum mechanics we must find a non-dissipative strange attractor because quantum mechanics is manifestly Hamiltonian, i.e., non-dissipative. This must have been the point where Palmer started getting interested in a dissipative quantum mechanics proposal by Nobel laureate 'tHooft [11]. The reader must recall that adding dissipation to quantum mechanics is a very tricky business because this would normally destroy everything, including any Lagrangian formulation [12]. However, it was again El Naschie who made some definite proposals in this direction and pointed out to 'tHooft that dissipative deterministic quantum mechanics is a concept homomorphic to non-dissipative non-deterministic classical mechanics [12].

The question is how did El Naschie reach this conclusion? The short answer is the VAK which we mentioned at the very beginning [4,8]. There is a widespread misconception that strange attractors exist only for dissipative systems. However, there are non-chaotic strange attractors studied for instance by Kapitaniak and El Naschie. Similarly there are things resembling strange attractors which exist generically in Hamiltonian systems (see Ref. [19] and the references therein). These are the vague attractors of Kolmogorov (VAK) which represent the vacuum fluctuations of E-infinity theory [13,14] and the sub structure of quantum mechanics [2,3,13,14]. A second misconception is that phase space, Hilbert space and spacetime are always essentially totally different things [15–17]. For instance while Hilbert space  $H(\infty)$  is basically a mathematical construction,  $E(\infty)$  is a physical spacetime [15,17]. Never the less, El Naschie demonstrated in various papers that  $H(\infty)$  could be viewed as almost as physical as  $E(\infty)$  and in fact  $E(\infty)$  could replace  $H(\infty)$  so that the invariant limit set of the VAK is the same as the limit set of the Klein or Möbius transformation of spacetime itself [2,22]. We should recall that at ultra high

energy, time is specialized similar to Euclidean quantum mechanics so that phase space and spacetime are identical. In addition at Planck energy we have only Planck masses, i.e., mini black holes which were discussed in this connection elsewhere [20–22].

There are many similarities in concept between Palmer's work and that of El Naschie's E-infinity theory. For instance Palmer starts with Borel normal numbers [18] and similarly El Naschie from a Borel set [2]. Number theoretical considerations are basic to Palmer's work and the same is true for El Naschie. Finally El Naschie uses the empty set to motivate the essential negative sign in his indistinguishability condition and similarly, Palmer relates the empty set to the paradoxes plaguing quantum mechanics [2,7,10,18]. Both El Naschie and Palmer appeals to Gödel theorems as well as Turing halting problem [2].

We may also recall in this context a friendly scientific dispute regarding strange but non-chaotic strange attractors in hydrodynamics between a Fellow of the Royal Society of London, Prof. Dowker and Prof. El Naschie which led to the rediscovery of the VAK with its golden mean KAM structure for quantum mechanics and high energy particle physics [19].

In conclusion we express our conviction that the combination between fractal-Cantorian spacetime theories and the dissipative and deterministic quantum mechanics proposals are right now paving the road to a complete understanding of quantum mechanics and thus quantum gravity [14,18].

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